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Fractal behaviour of flow of inhomogeneous fluids over smooth inclined surfaces

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Abstract

Patterns formed by the flow of an inhomogeneous fluid (suspension) over a smooth inclined surface were studied. It was observed that fractal patterns form. There exists a threshold angle for the inclination above which global fractal patterns are formed. This angle depends on the particle size of the suspension. We observed that there are two fractal dimensions for these patterns, depending on the area from which the pattern is extracted. If the pattern is taken from the top which only consists of the beginning steps of the pattern forming, one finds two fractal dimensions, i.e. 1.35–1.45 and 1.6–1.7, in which the first one is dominant while, if the entire pattern is taken, then a fractal dimension 1.6–1.7 is observed. The first fractal dimension belongs to the class of the flow of water over an inhomogeneous surface and the second one corresponds to the river network. This may imply that both universality classes are present. However, here disorder is present in the fluid and is transferred to the surface.

1. Introduction

The flow of fluid in a random media as an instance of collective nonlinear transport with strong disorder [1–3] has attracted much attention. Macroscopic transport occurs only when the driving force exceeds a threshold magnitude. Near the threshold, there is some evidence of critical behaviour, including diverging correlation length. Many interesting effects may be expected such as scaling relations between the critical exponent above and below the threshold, etc. Examples of this behaviour can be seen in the patterns formed on the window pane during rain [1]. In all these examples, the environment is strongly disordered but, through this environment, a simple body (flux line in the superconductors [2]), a homogeneous fluid (rain on a dirty rough surface) flows [1] or a porous medium is invaded by a nonwetting fluid [3]. In contrast, we are concerned with systems in which disorder is present within the moving fluid and the environment is homogeneous: suspensions made from yoghurt and water, talcum

Figure 1. Digitized pattern of the flow of the yoghurt–water suspension over an inclined surface. The white areas are precipitated yoghurt, the black areas are washed by the flow of water. This is above the threshold angle in which we can see many global fractal trees.

powder and water, and flour and water flowing down over an inclined glass surface. The surface is clean and the fluids do not erode it.

2. Experimental procedures

The experiments were carried out with three suspensions: yoghurt and water, talcum powder and water, and flour and water, in controlled proportions. The main advantage of these mixtures is their cheapness and availability, all of them have been bought from the supermarket. The ratio of the mixture was kept fixed during all the experiments. The volume ratio of yoghurt to water (for example) was 100–150. For sample preparation first we stirred the 100 cm^3 yoghurt perfectly, then we added 150 cm^3 water to it, and then mixed them by stirring. Flour and talcum powder can be mixed more easily.

A device was built with a container filled by the fluid, which is then rotated to allow for uniform pouring of the fluid onto the top rim of an inclined plate. The top surface of the plate, on which the fluid is poured, is covered by a clean plate of glass A calibrated plate allows the glass plate to be adjusted to the desired angle. This glass plate can be removed from the device and can be cleaned by water and some detergent and then with pure water. After that we dry the glass plate carefully either by a soft cotton cloth or a hairdryer.

Once the fluid is poured down the glass inclined surface, the precipitation patterns, left by the fluid, have obvious fractal features. These patterns were registered and digitized by

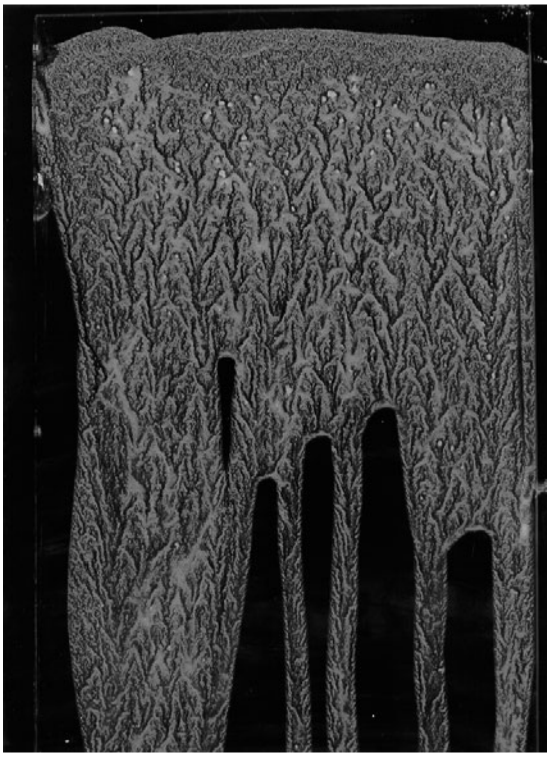

Figure 2. Pattern formation below the threshold angle. There is no global fractal tree connecting the top to the bottom of the flow.

applying a scanner and a computer. An example of the digitized pattern is shown in figure 1. The fractal pattern formed by the downward flow over the glass inclined surface is evident. We observe that there exists a critical angle of inclination. If the angle of the inclined plate is below this threshold, the fractal pattern is not global, i.e. there does not exist even one fractal tree connecting the top to the bottom of the flow. Above the threshold angle there exist many global fractals, see figure 1. We found this angle to be between 10° and 12° for yoghurt, 8° and 10° for talcum powder and 34° and 36° for flour. The threshold angle is where one observes the first global fractal. Figure 1 shows pattern formation above the threshold angle and figure 2 shows pattern formation below the threshold angle. Here we report the scaling properties of the system. Our observations are consistent with the theoretical results of the flow of fluid over rough surfaces [4, 5] at the beginning of pattern formation, i.e. on the top of the plate, and also consistent with the river network for the whole pattern.

A frequency distribution of particle size is shown in figures 3–5. We observe that the particle size of yoghurt is sharply defined and is around 50 μm. Figures 4 and 5 show the particle size distribution for talcum powder and flour, respectively. The particle size for talcum powder is about 25 μm and for flour is approximately 18 μm. Specifications of these suspensions obtained through measurements by a particle size analyser are given in table 1.

3. Image analysis

After importing a digitized pattern, the fractal dimension is measured using three different methods.

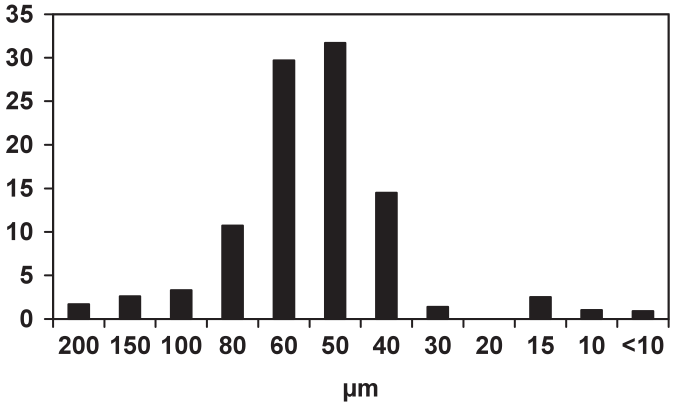

Figure 3. Frequency distribution of particle diameters for yoghurt.

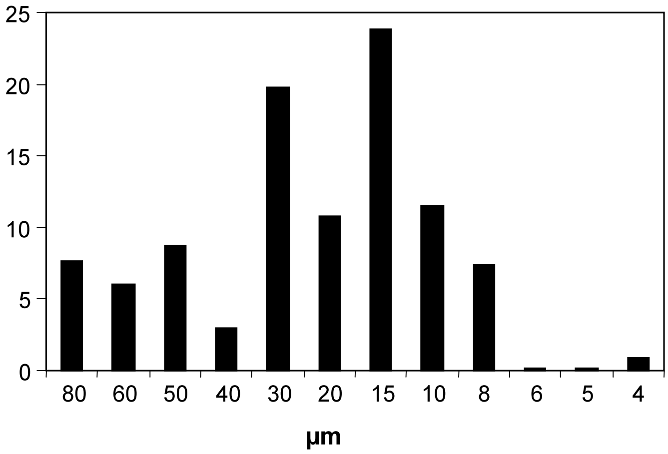

Figure 4. Frequency distribution of particle diameters for flour.

Table 1. Particle specifications.

Suspension	Median diameter (μm)	Modal diameter (μm)	Surface area ($m^2\ g^{-1}$)
Yoghurt	49.37	48.28	1.14
Flour mix	17.75	12.29	0.28
Talcum mix	24.75	24.4	0.22

Method 1. The mass of a particular stream, m, is related to its downward length of flows by [1]

$$m(\ell) = \ell^{d_f}.$$

Here the mass refers to the total surface area of a stream which is measured in the computer from the digitized patterns by counting all the pixels which cover the stream under consideration (this is done by our computer software). ℓ is its length. In this method several patterns are taken from a fractal tree at different scales and for each of them $m(\ell)$ and ℓ are calculated and

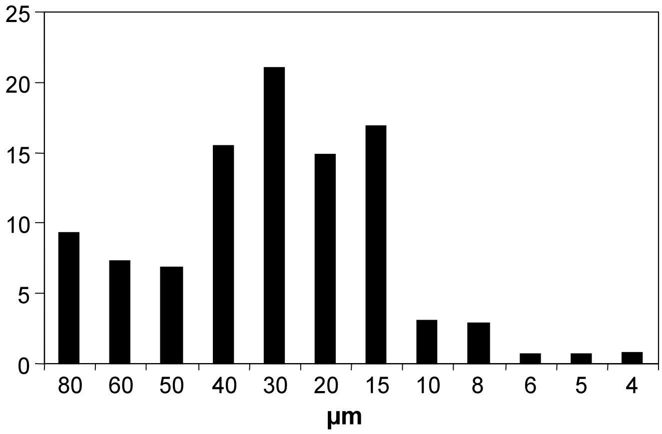

Figure 5. Frequency distribution of particle diameters for talcum powder.

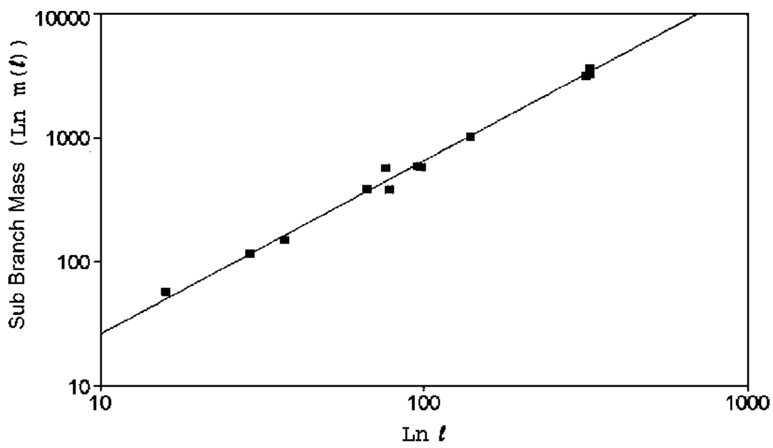

Figure 6. The fractal dimension of the yoghurt mix poured onto an inclined surface, where m is the surface area of the subbranch and ℓ is its length.

plotted. The exponent d_{f} is then calculated. The average of all curves yields the final exponent d_{f}.

Figure 6 shows one of these graphs for yoghurt and its slope. In this method d_{f} is found to be in the range 1.35–1.45 for the top of the pattern.

Method 2. The total mass of stream, M, scales with the total length as

$$M(L) \sim L^{d_{\mathrm{f}}}.$$

This method is essentially the same as the first method except that subbranches are not taken but the whole fractal tree is used. In a log–log diagram (figure 7) these points form a straight line whose slope is the fractal dimension of the patterns. With this method the fractal dimension obtained for yoghurt was $d_{\mathrm{f}} = 1.40 \pm 0.05$ for the top of the fractal pattern.

Method 3. The last method is the correlation function method [1, 6], which is defined as follows:

$$C(r) \equiv \frac{1}{N} \sum_{i} \rho(\vec{r})\rho(\vec{r}_c + \vec{r}). \tag{1}$$

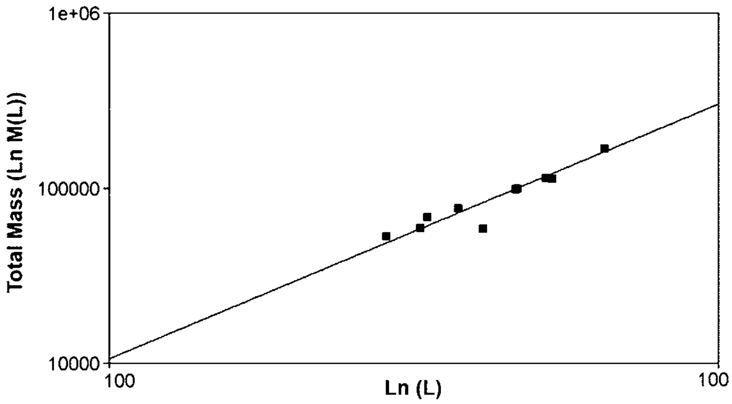

Figure 7. The fractal dimension through the scaling of mass with length.

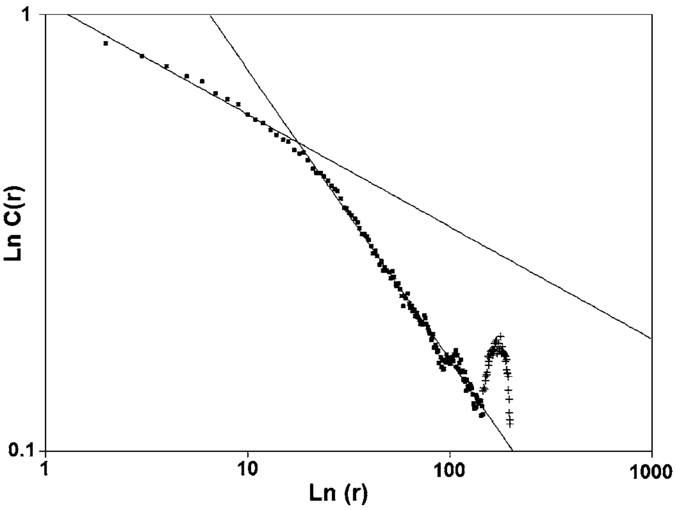

Figure 8. The correlation function for the downward direction of yoghurt, for the top portion of the pattern. For the mid-section $\alpha = 0.6$, while for the beginning section $\alpha = 0.3$. This corresponds to fractal dimensions of 1.4 and 1.7, respectively.

Here ρ is the local density. If the point of location $\vec{r}$ belongs to the structure, $\rho(r) = 1$ and if it does not belong to it, then $\rho(r) = 0$. N is the total number of points used for calculating the correlation function. The function $C(\vec{r})$ represents the expectation value that the two points belong to the pattern. For isotropic fractal patterns we expect $C(r)$ to depend not on direction, but only on the distance, hence $C(\vec{r}) = C(r)$.

In the present case the horizontal and vertical directions clearly differ. Therefore we expect that $\vec{r}$ has to be parallel to the direction of downward flow.

The correlation function satisfies a power law of the form

$$C(r) \sim r^{-\alpha}. \tag{2}$$

The mass of the stream $m(\ell)$ can be expressed in terms of the correlation function, from which it follows that the fractal dimension is [6]

$$d_{\mathrm{f}} = 2 - \alpha. \tag{3}$$

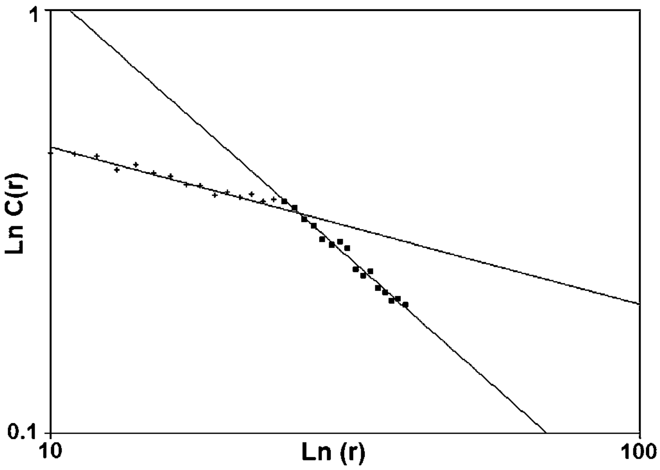

Figure 9. Correlation function for flour chosen from the top of the pattern. The fractal dimension for the mid-section is 1.34 and for the beginning section it is 1.7.

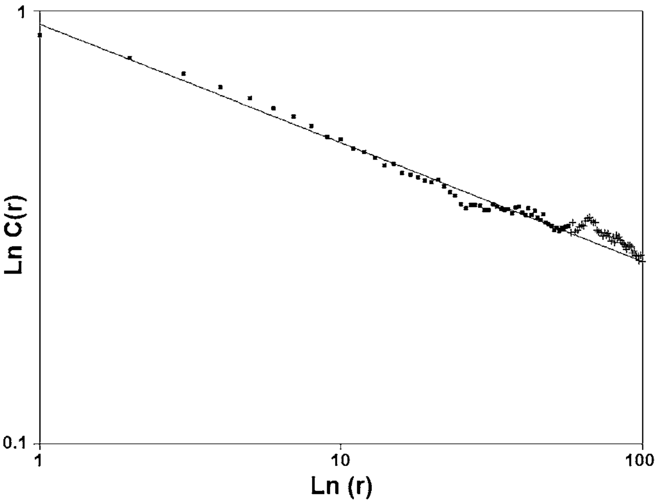

Figure 10. Correlation function of yoghurt for the whole pattern. The fractal dimension is 1.7.

This relation is the same as the one given by Vicsek, except that it is anisotropic. Anisotropy does not alter the scaling relation as noted in [1]. Figure 8 plots $C(r)$ versus r for the downward flow of the yoghurt suspension, in which the top part of the pattern was chosen.

The curve has two exponents in two regions, one at the beginning of the curve and the other in the mid-section. For the mid-section $\alpha = 0.6$, while for the beginning section $\alpha = 0.3$. This corresponds to fractal dimensions of 1.4 and 1.7, respectively.

Figure 9 shows the same result for flour. As can be seen in figure 9, we have two different slopes in the correlation function curve. The fractal dimension is 1.7 for the first section and 1.4 for the second section.

Figures 10, 11 and 12 show the correlation functions for the yoghurt, talcum powder and flour suspensions, respectively, when the whole pattern was chosen for analysis. The fractal dimension for all of them lies in the range 1.6–1.75.

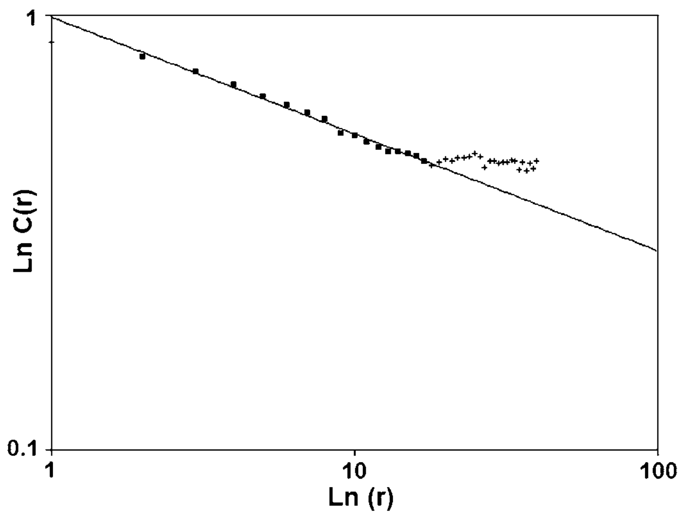

Figure 11. Correlation function for the entire pattern of the talcum powder. The fractal dimension is 1.73.

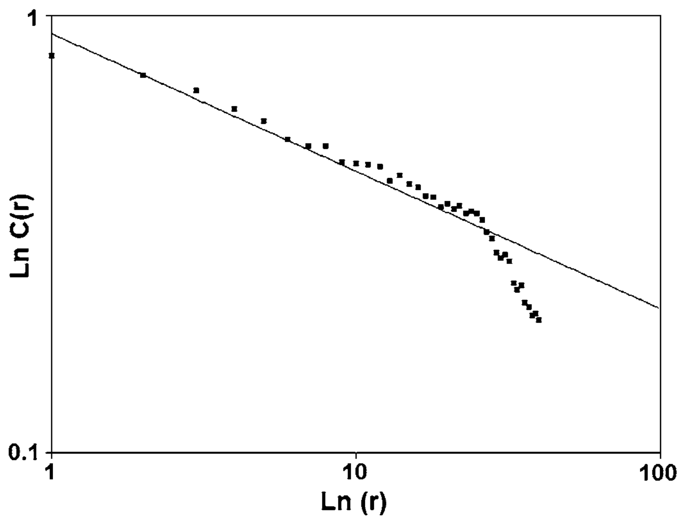

Figure 12. The correlation function of flour, all points included, giving a fractal dimension of 1.69.

4. Conclusion

Narayan and Fisher [4] proposed a model for the nonlinear behaviour of the flow of fluid over a rough random surface. In this model it is assumed that the disorder is strong enough to break the flow into several channels. As the inclination angle of the surface is slowly increased, the fluid collects into lakes with more depth at the lower end of the lake. Any further increase in the inclination will cause the fluid to find a way to the adjacent lakes lower down, forming clusters of lakes. Therefore, as the inclination angle of the surface becomes larger, the length of the clusters formed increases, reaching an infinite value at a critical threshold angle. Below this limit it is possible to see many isolated clusters which are totally disconnected from the flow. At threshold, there exists at least one flowing river from top to bottom (e.g. at least one cluster whose correlation length is infinite).

Mean field theory also predicts the value $4/3$ for the fractal dimension of this kind of grid [4].

The system under study is different from the model of Narayan and Fisher, because the disorder is in the fluid not in the surface. In spite of this difference, the measured fractal dimension for the top of the pattern, i.e. for the early stages of the formation of the pattern, is 1.40 ± 0.05, which is in good agreement with the predictions of the model by Narayan and Fisher [4] and the mean field theory [4], and also with experimental observations in an experiment in which the movement of rain over an inclined dirty surface has been studied [1]. In this experiment the fractal dimension was measured to be 1.37 ± 0.05, which is consistent with our measurement for the early stages of pattern formation.

On the other hand, for the whole of the pattern our measured fractal dimension is 1.6–1.75 which is in good agreement with the fractal dimension for the river networks [1].

Apparently the system studied before the joining of the streams may be in the same class of universality as the model of Narayan and Fisher [4], whereas after joining the streams we may have a river network in which the fractal dimension is 1.6–1.7 [1].

Acknowledgments

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